EPFL

Arrays

Antennas

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Synopsis

- Why a theory on array antennas?
- Mutual coupling
- The array factor
- Some examples
  - Linear equidistant array
  - Linear phased array
  - Schelkunoff synthesis
  - The binomial array

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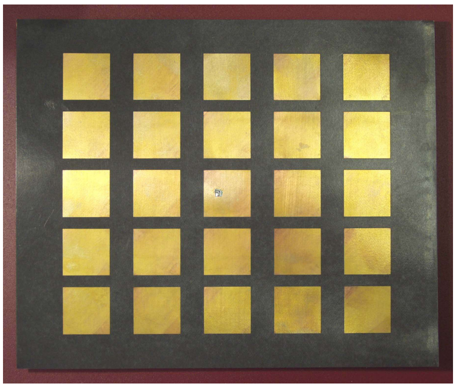
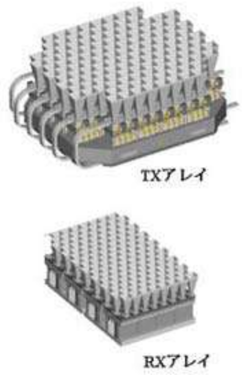
Examples of array antennas





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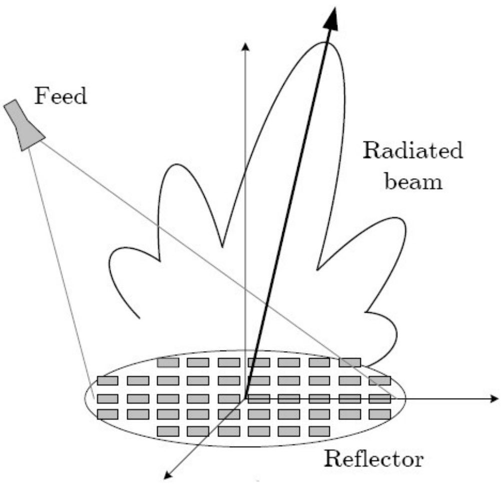
Reflectarray



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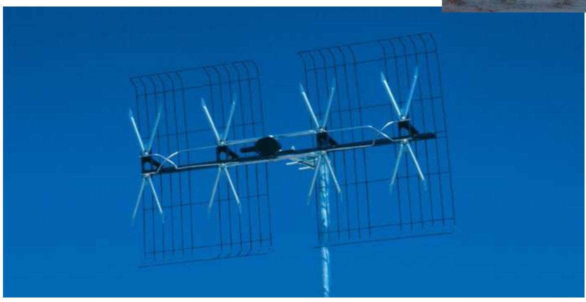
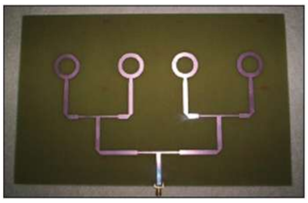
Array antennas





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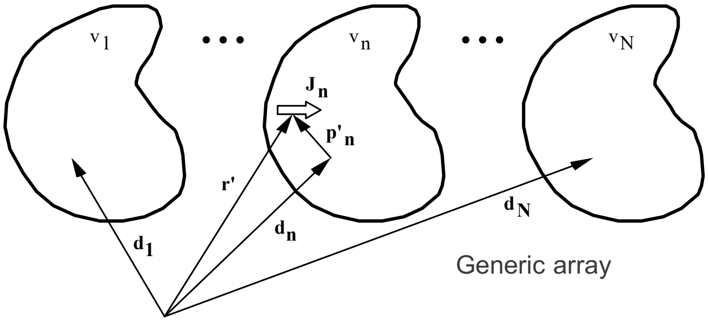
Mutual coupling

Mutual coupling



Generic array

- By definition, all elements are identical

r' : denotes any point, in this case located on element #n

d_n : denotes the origin of the coordinate system local to element #n

p'_n : denotes any point on element #n with respect to its local coordinate system

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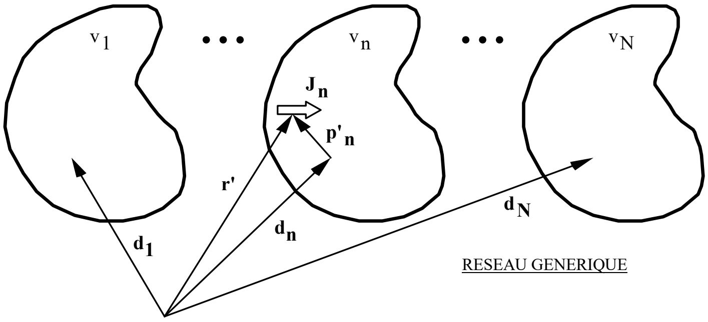
Mutual coupling



RESEAU GENERIQUE

Current density on one element : $J(r') = J(d_n + p'_n) = J_n(p'_n)$

$$J_1(p'_1) = J_2(p'_2) = J_n(p'_n) \ ???$$

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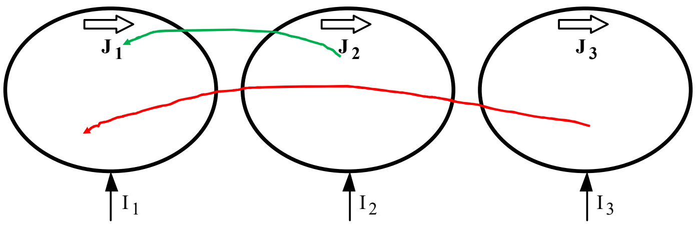
EPFL Mutual coupling



$$I_1 = I_2 = I_3$$

$$J_1(p'_1) = J|_{I_1}(p'_1) + J|_{I_2}(p'_1) + J|_{I_3}(p'_1)$$

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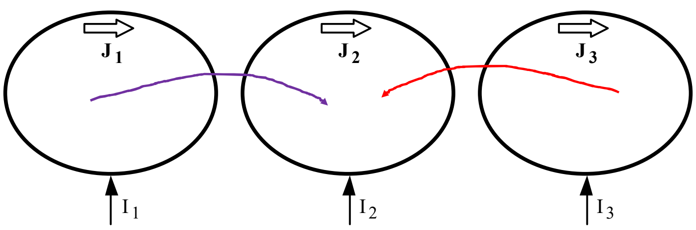
EPFL Mutual coupling



$$I_1 = I_2 = I_3$$

$$J_2(p'_2) = J|_{I_1}(p'_2) + J|_{I_2}(p'_2) + J|_{I_3}(p'_2)$$

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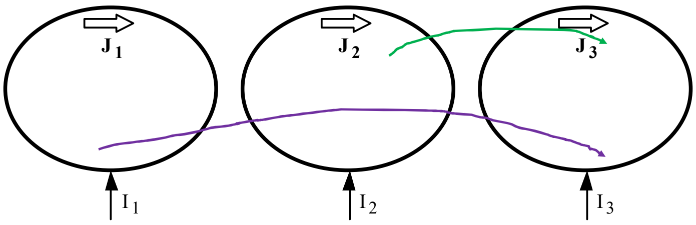
Mutual coupling



$$I_1 = I_2 = I_3$$

$$\boldsymbol{J}_3(\boldsymbol{p}'_3) = \boldsymbol{J}\big|_{I_1}(\boldsymbol{p}'_3) + \boldsymbol{J}\big|_{I_2}(\boldsymbol{p}'_3) + \boldsymbol{J}\big|_{I_3}(\boldsymbol{p}'_3)$$

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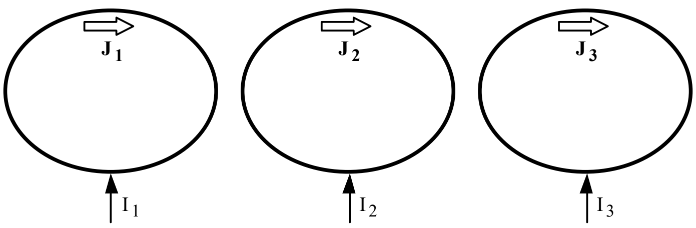
Mutual coupling



$$I_1 = I_2 = I_3$$

$$\boldsymbol{J}_1(\boldsymbol{p}'_1) \neq \boldsymbol{J}_2(\boldsymbol{p}'_2) \neq \boldsymbol{J}_3(\boldsymbol{p}'_3)$$

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The influence between elements is the mutual coupling

Mutual coupling

- Hard to take into account as not trivial to obtain
- Often very weak
- Neglected in first approximation

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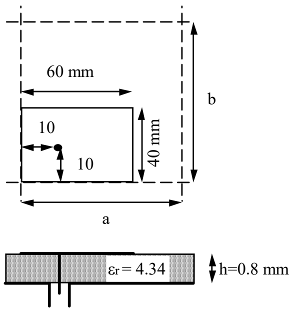
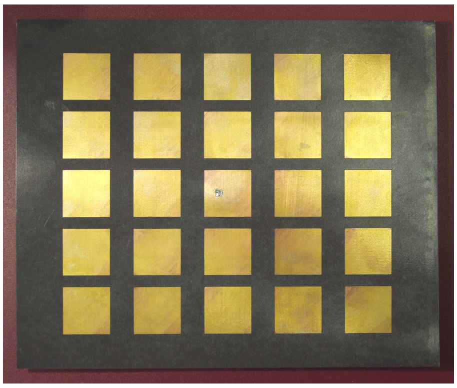
Hypothesis: mutual coupling betweeen element is equal to zero

How true is it ? An example





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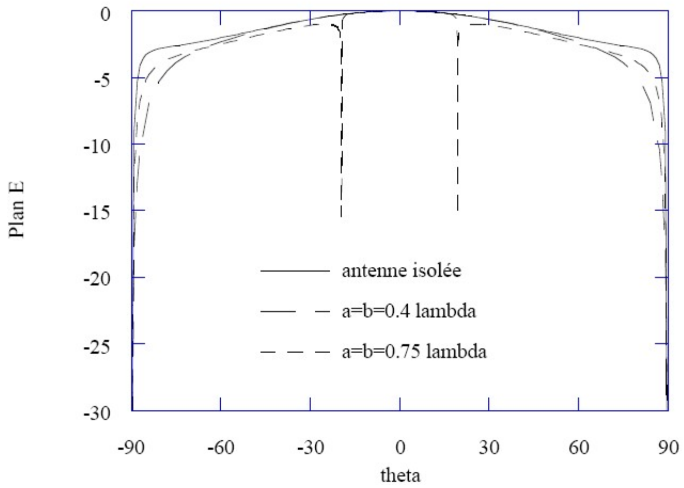
How true is it ?



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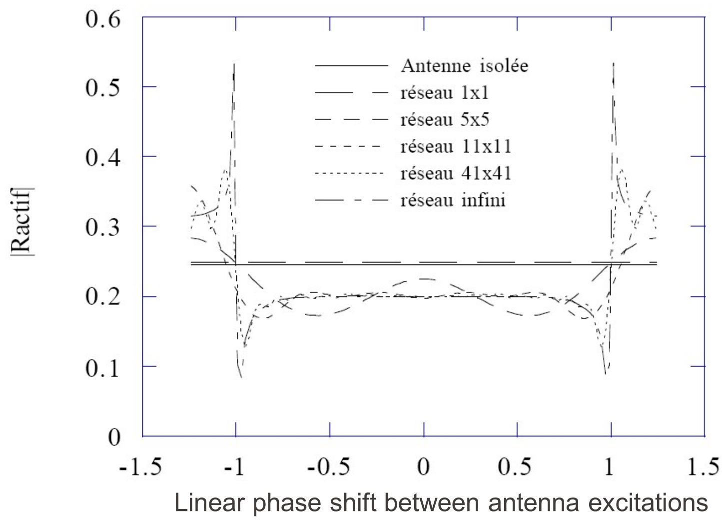
How true is it?

All antenna elements are excited with the same amplitude and a linear phase shift



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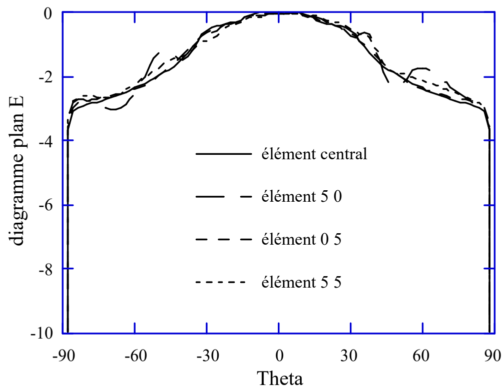
How true is it?



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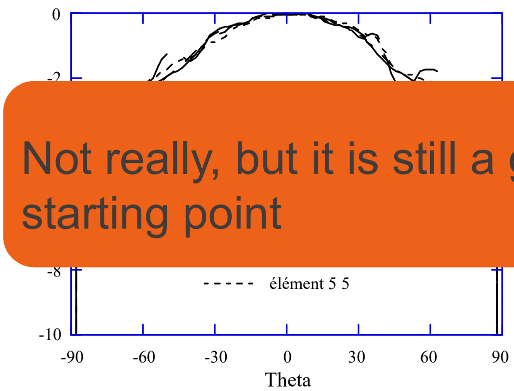
How true is it ?



Not really, but it is still a good starting point

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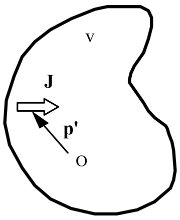
Zero mutual coupling hypothesis

Isolated element used as reference



The reference current is given by

$$J(p')$$

$$J(d_n + p'_n) = J_n(p'_n) = I_n J(p')$$

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with

$$\frac{J_n(p'_n)}{J_m(p'_m)} = \frac{I_n}{I_m}$$

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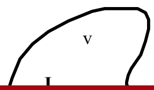
Zero mutual coupling hypothesis

Isolated element used as reference



Le courant sur l'élément

Hypothesis used in everything that follows

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with

$$\frac{J_n(p'_n)}{J_m(p'_m)} = \frac{I_n}{I_m}$$

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The array factor

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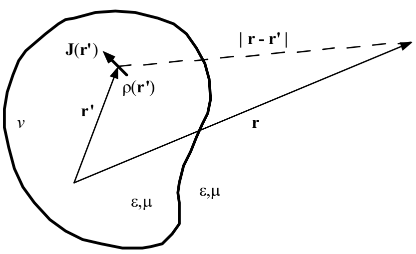
The array factor

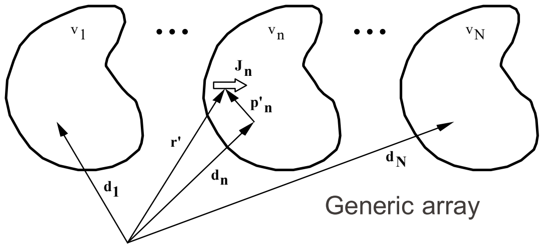
The vector potential of the isolated element is given by



$$A(r) = \frac{\mu}{4\pi}\frac{e^{-jkr}}{r} f(\theta,\varphi)$$

$$f(\theta,\varphi) = \int_v dv' J(r') e^{jke_r \cdot r'}$$

EPFL The array factor



$$f(\theta,\varphi)=\sum_n \int_{v_n} dv'\boldsymbol{J}(\boldsymbol{r}')e^{jk\boldsymbol{e}_r\cdot\boldsymbol{r}'}$$
$$=\sum_n e^{jk\boldsymbol{e}_r\cdot\boldsymbol{d}_n}\int_{v_n} dv'\boldsymbol{J}(\boldsymbol{d}_n+\boldsymbol{p}'_n)e^{jk\boldsymbol{e}_r\cdot\boldsymbol{p}'_n}$$
$$=\left[\int_{v_e} dv'\boldsymbol{J}(\boldsymbol{r}')e^{jk\boldsymbol{e}_r\cdot\boldsymbol{p}'}\right]\left[\sum_n I_n e^{jk\boldsymbol{e}_r\cdot\boldsymbol{d}_n}\right]$$

v_e: Volume of one element

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EPFL The array factor

$$f(\theta,\varphi)=\underbrace{\left[\int_{v_e} dv'\boldsymbol{J}(\boldsymbol{r}')e^{jk\boldsymbol{e}_r\cdot\boldsymbol{p}'}\right]\left[\sum_n I_n e^{jk\boldsymbol{e}_r\cdot\boldsymbol{d}_n}\right]}_{\boldsymbol{f_e}(\theta,\varphi)\,AF(\theta,\varphi)}$$

$$AF(\theta,\varphi)=\sum_n I_n e^{jk\boldsymbol{e}_r\cdot\boldsymbol{d}_n}$$

Array factor:

Depends on the position and excitation of the array elements

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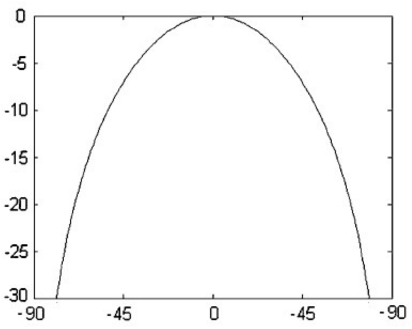
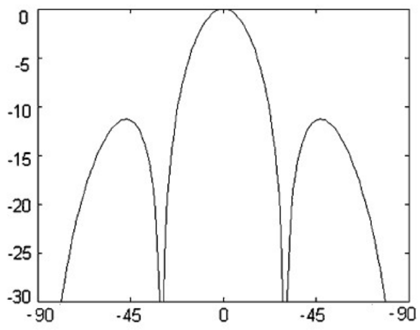
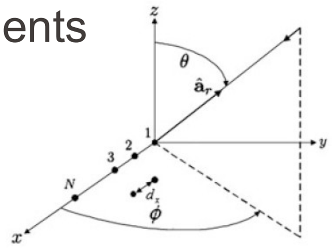
Array factor: examples



2 elements



4 elements



Spacing: λ/2

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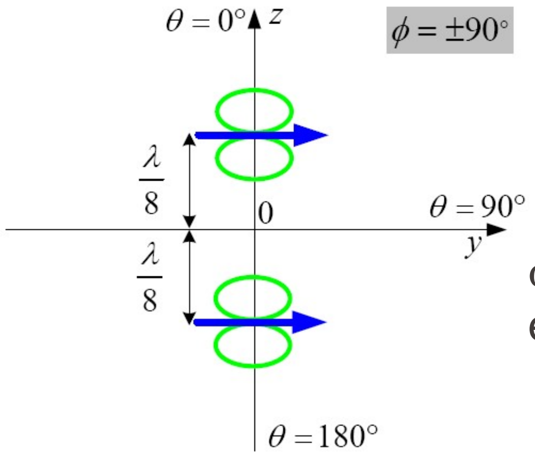
Example: Array factor and radiation pattern



$$AF = \cos\left(\frac{kd\cos\theta + \alpha}{2}\right)$$

α: phase shift between the excitation of the two dipoles

2 Hertzian dipoles excited with the same amplitude

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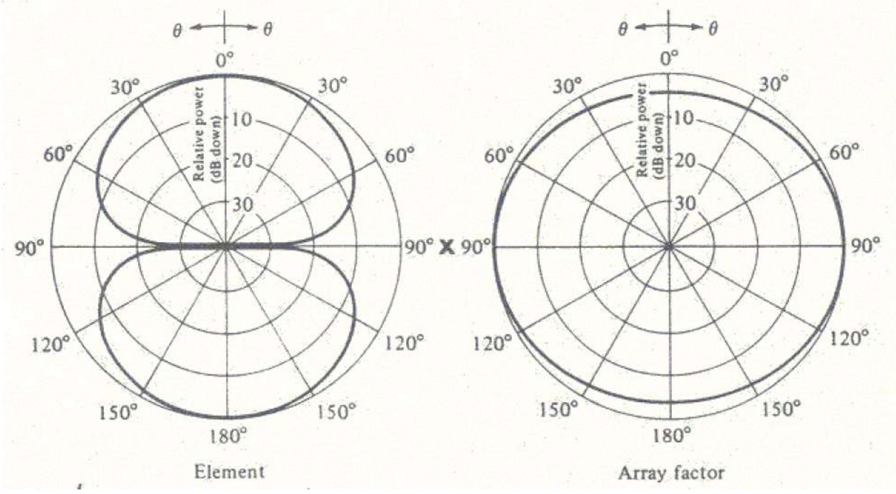
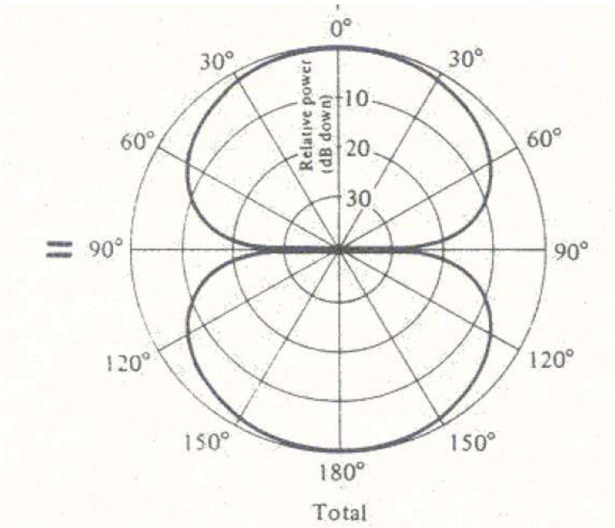
α=0°

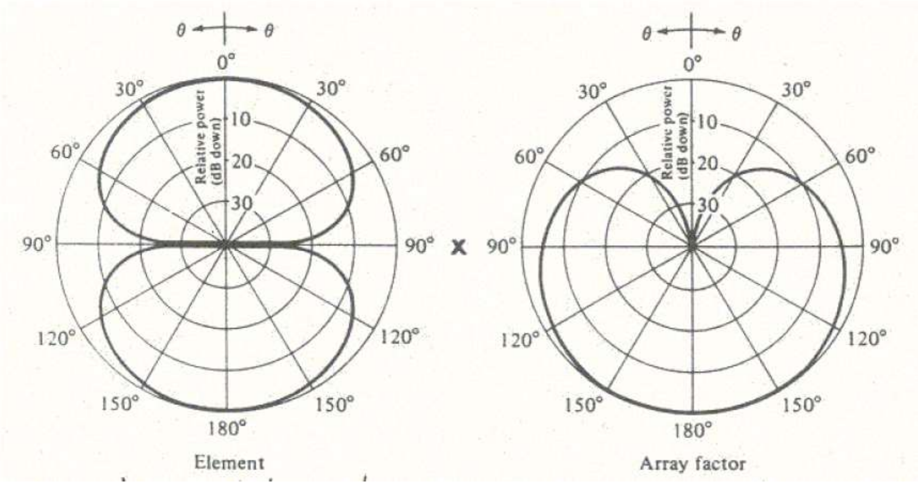


α=0°



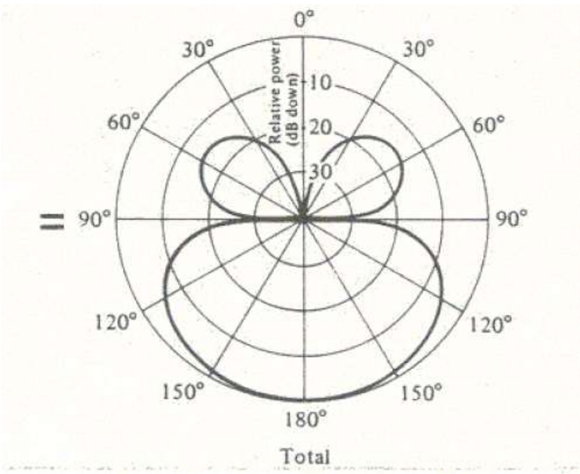
C.A. Balanis, Antenna Theory, Analysis and Design, Harper and Row, 1982

EPFL α=90°



C.A. Balanis, Antenna Theory, Analysis and Design, Harper and Row, 1982

EPFL α=90



C.A. Balanis, Antenna Theory, Analysis and Design, Harper and Row, 1982

The array factor

For an isotropic antenna $|\mathbf{f_e}(\theta,\varphi)|=1$

In this case, the array factor is also the radiation pattern of the dipole

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The array factor

$$\boldsymbol{f}(\theta,\varphi)=\underbrace{\left[\int_{v_e} dv'\boldsymbol{J}(\boldsymbol{r}')e^{jk\boldsymbol{e}_r\cdot\boldsymbol{p}'}\right]\left[\sum_n I_n e^{jk\boldsymbol{e}_r\cdot\boldsymbol{d}_n}\right]}_{\boldsymbol{f_e}(\theta,\varphi)\,AF(\theta,\varphi)}$$

$$AF(\theta,\varphi)=\sum_n I_n e^{jk\boldsymbol{e}_r\cdot\boldsymbol{d}_n}$$

Array factor:

Depends on the position and excitation of the elements

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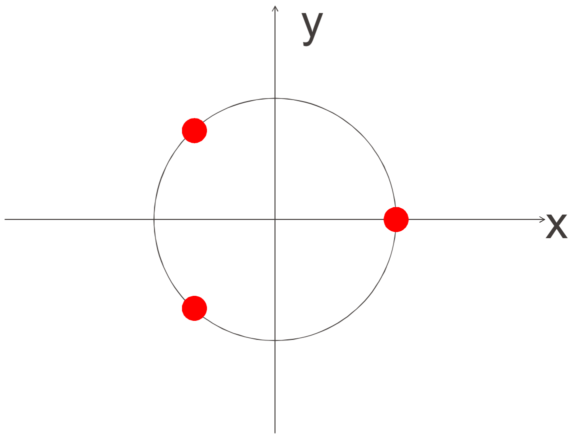
Example: Circular array



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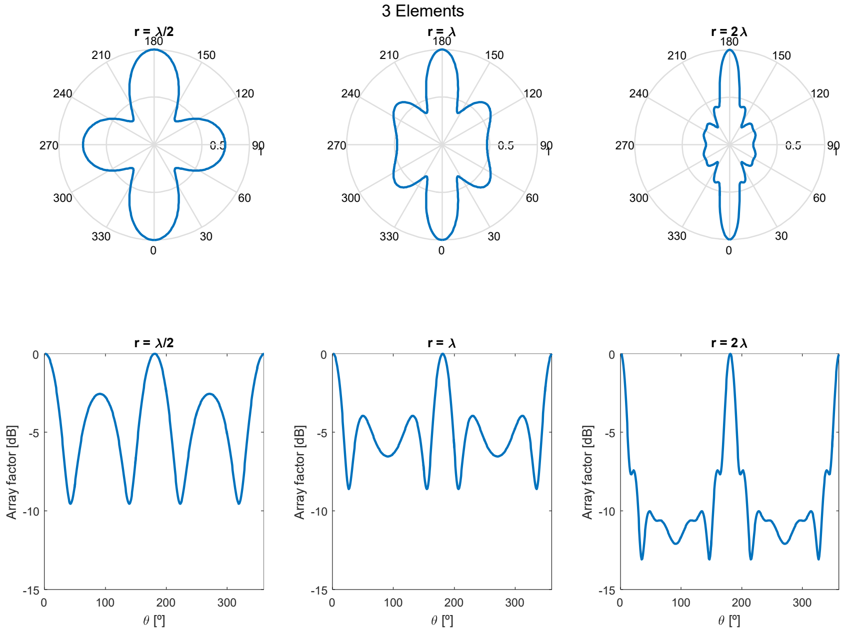

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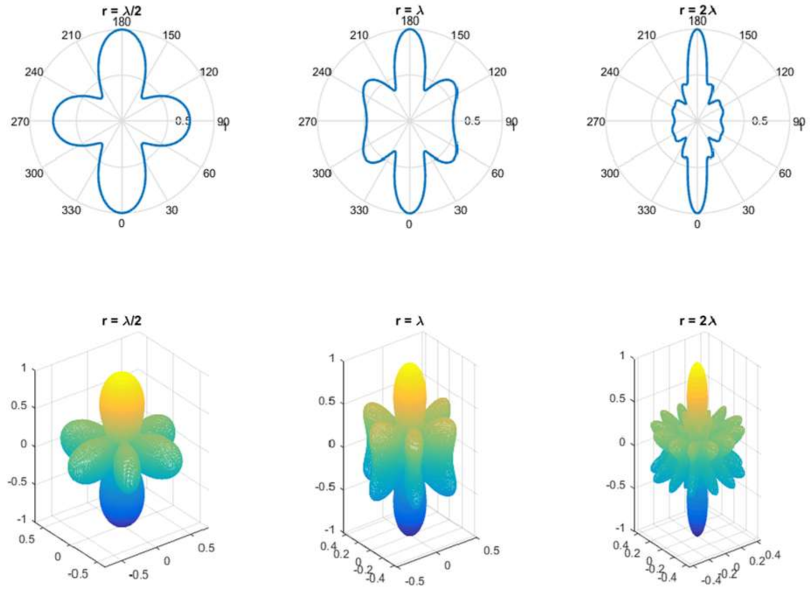
3 Elements



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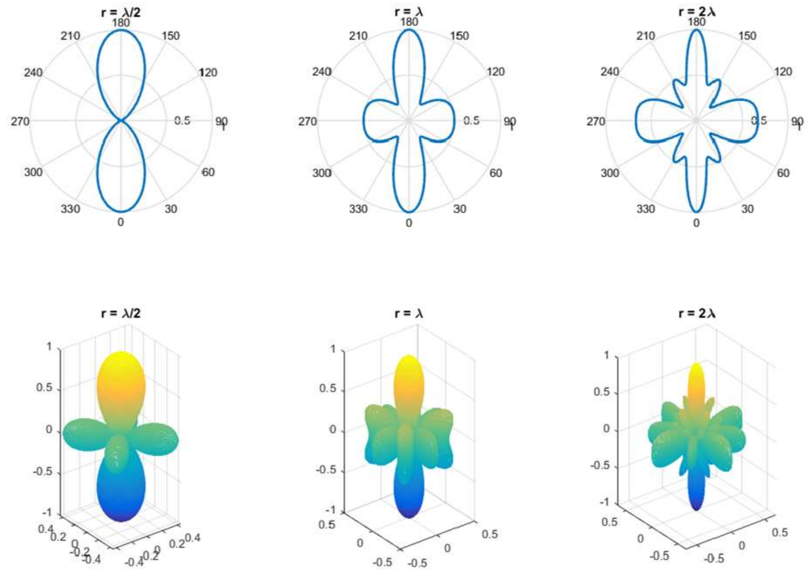
4 Elements



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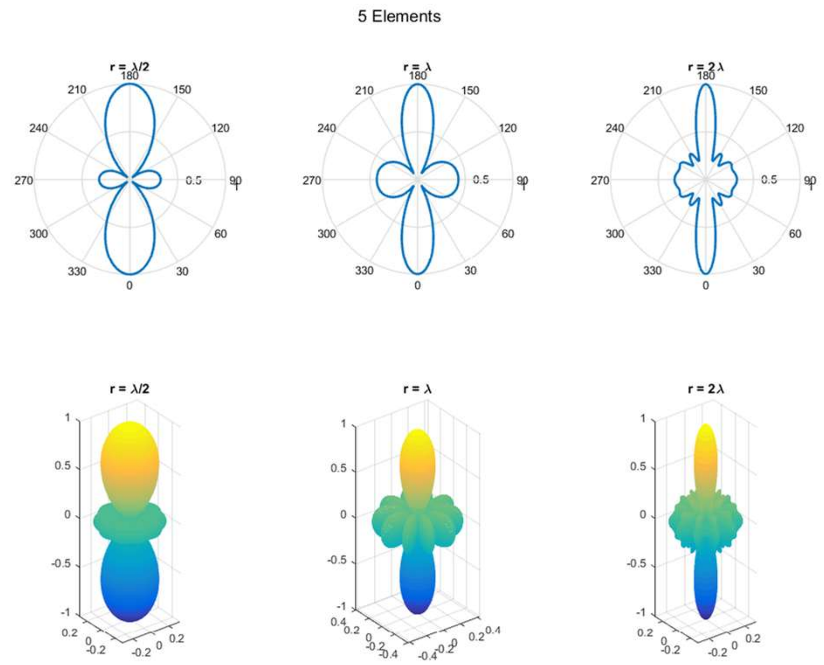

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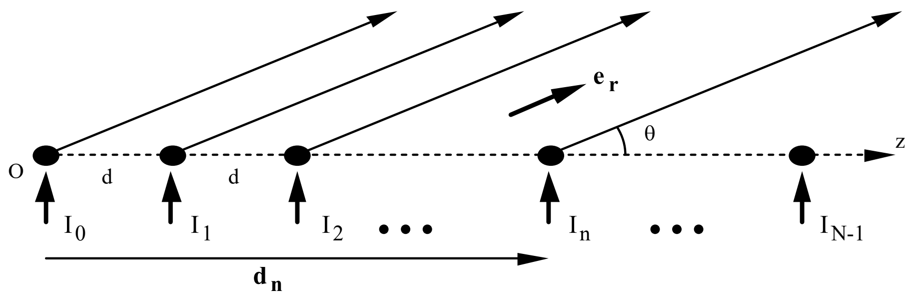
Equidistant linear arrays



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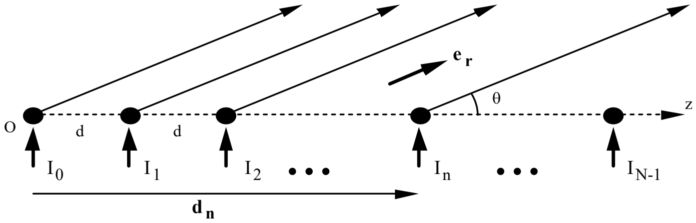
- axis z= Axis of the array
- Elements are numbered from 0 to N-1
- The origin is placed at element 0

$$AF(\theta,\varphi) = \sum_n I_n e^{jk\mathbf{e}_r \cdot \mathbf{d}_n}$$

Equidistant linear arrays



$$AF(\theta,\varphi)=\sum_n I_n e^{jk\mathbf{e}_r\cdot \mathbf{d}_n}$$

$\mathbf{d_n}$ = nd **ez**

er • $\mathbf{d_n}$ = nd cos θ

Symmetry of revolution, thus AF(θ) depends only on θ

$$AF(\theta,\varphi)=\sum_{n=0}^{n=N-1} I_n e^{jnkd\cos\theta}$$

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Equidistant linear arrays

$$I_n = A_n e^{j\alpha_n}$$

$$AF(\theta,\varphi)=\sum_{n=0}^{n=N-1} A_n e^{(jnkd\cos\theta+\alpha_n)}$$

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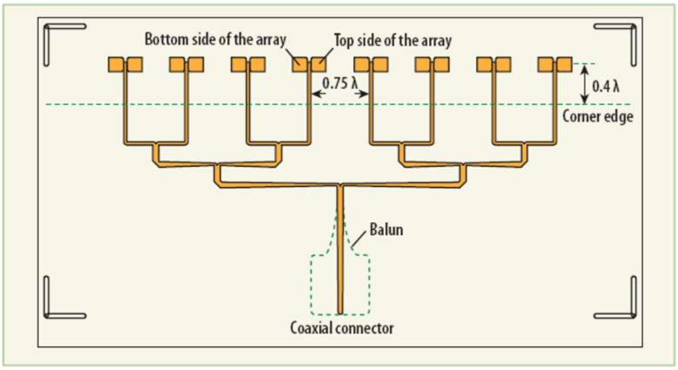
Equidistant linear arrays with linear phase shift

$$I_n = A_n e^{jn\alpha} \quad AF(\theta,\varphi) = \sum_{n=0}^{n=N-1} A_n e^{(jn(kd\cos\theta+\alpha))} = \sum_{n=0}^{n=N-1} A_n e^{(jn\psi)}$$



1. This layout represents the printed patch array antenna with feed network and balun.

$$\psi = kd\cos\theta + \alpha$$

Easy to control by adjusting the length of the lines

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Equidistant linear arrays with linear phase shift

Variable ψ depends on:

- The frequency (k)
- The interelement distance (d)
- Beam tilting angle (θ)
- The phase shift between two elements (α)

The array factor is periodic in ψ
Do not mix up ψ and θ

$\theta[-\pi,\pi]$ <=> ψ [-kd+α , kd+α] : visible space of ψ

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Equidistant linear arrays with linear phase shift

$$\psi = kd\cos\theta + \alpha$$

$$\cos\theta = \frac{\psi}{kd} - \alpha$$

If kd<π. ψ may not take all between -π et π,
Which correspond to the visible space

In this case, we need to use imaginary values for cos θ to cover an entire period of ψ. We talk about the invisible region in these cases.

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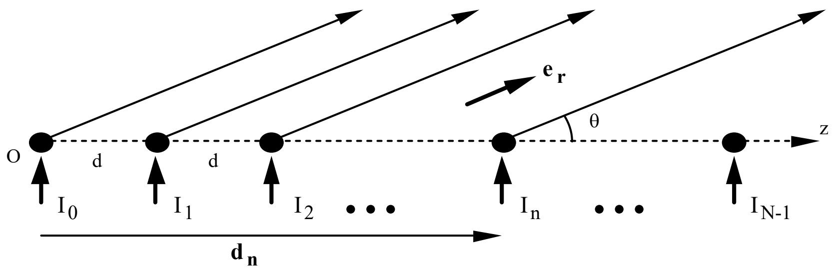
Equiamplitude array with linear phase shift



$$I_n = e^{jn\alpha} \qquad \psi = kd\cos\theta + \alpha$$

$$AF(\theta,\varphi) = \sum_{n=0}^{n=N-1} A_n e^{(jn(kd\cos\theta+\alpha))} = \sum_{n=0}^{n=N-1} A_n e^{(jn\psi)} = \sum_{n=0}^{n=N-1} e^{(jn\psi)} = \frac{e^{jN\psi}-1}{e^{j\psi}-1}$$

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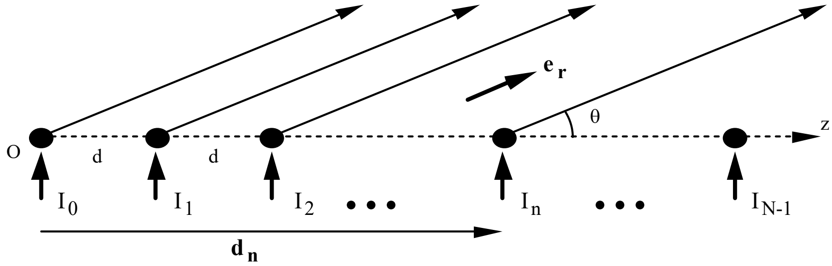
Equiamplitude array with linear phase shift



$$I_n = e^{jn\alpha} \qquad \psi = kd\cos\theta + \alpha$$

$$AF(\theta,\varphi) = \frac{e^{jN\psi}-1}{e^{j\psi}-1} = \frac{\sin\left(N\frac{\psi}{2}\right)}{\sin\left(\frac{\psi}{2}\right)} e^{j\left(N-1\right)\frac{\psi}{2}}$$

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Normalized array factor

$$\left|AF(\theta,\varphi)\right| = \left|\frac{\sin\left(N\frac{\psi}{2}\right)}{\sin\left(\frac{\psi}{2}\right)}\right| \qquad NAF(\theta,\varphi) = \left|\frac{\sin\left(N\frac{\psi}{2}\right)}{N\sin\left(\frac{\psi}{2}\right)}\right|$$

Indeed, the array pattern concerns the relative values of the fields

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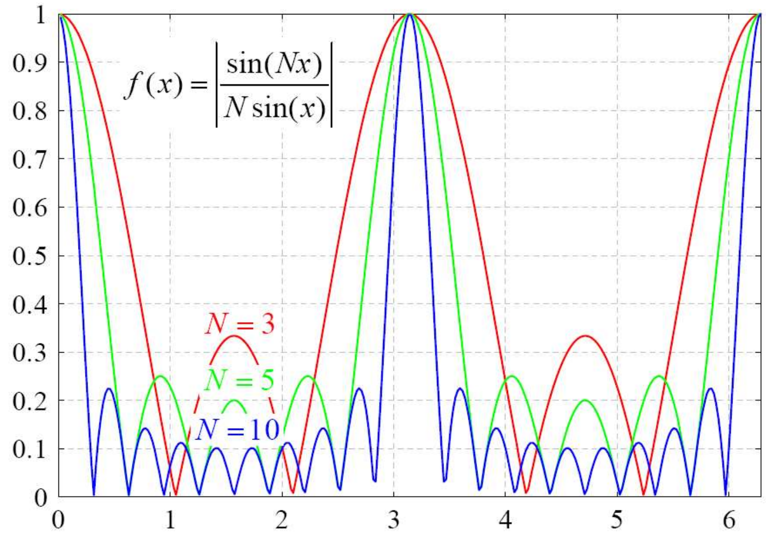
Normalized array factor



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Normalized array factor for an equi-amplitude array with linear phase shift

$$NAF(\theta,\varphi) = \left| \frac{\sin\left(N\frac{\psi}{2}\right)}{N\sin\left(\frac{\psi}{2}\right)} \right|$$

- Direction for zero radiation for $N\psi/2=+\pi$ or $-\pi$
- Main beam: between the zeroes
- Beamwidth: $4\pi/N$

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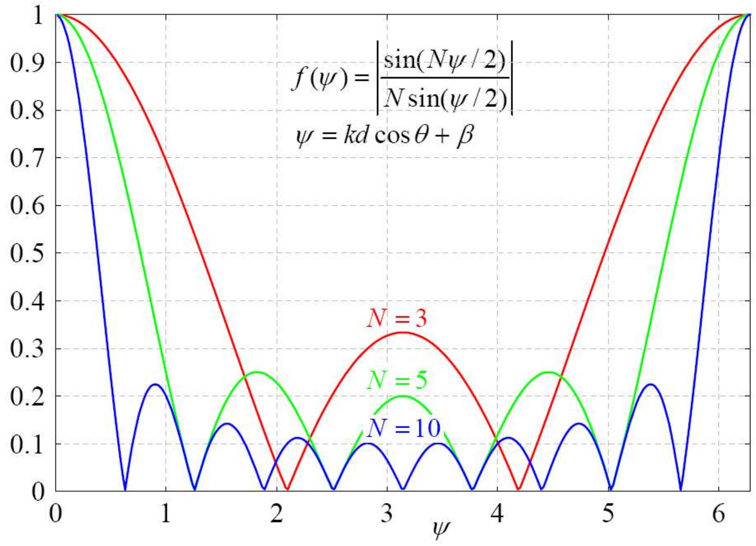
Sidelobe level



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Normalized array factor for an equi-amplitude array with linear phase shift

$$NAF(\theta,\varphi)=\left|\frac{\sin\left(N\frac{\psi}{2}\right)}{N\sin\left(\frac{\psi}{2}\right)}\right|$$

- If N is large, the sidelobe reaches its max. for

  ψ=3π/2
- Max level of teh sidelobe:

$$\frac{1}{N\sin\left(\frac{3\pi}{N}\right)}$$

When n goes to infinity,

This value tends to:

$$\frac{2}{3\pi}=-13=dB$$

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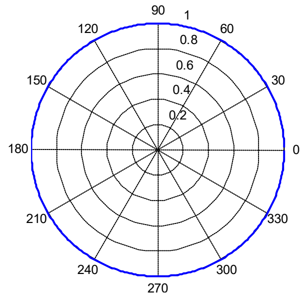
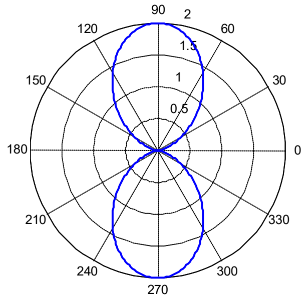
EPFL Examples



Linear array, N=1, d=0.5λ



Linear array, N=2, d=0.5λ

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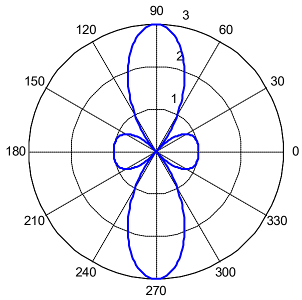
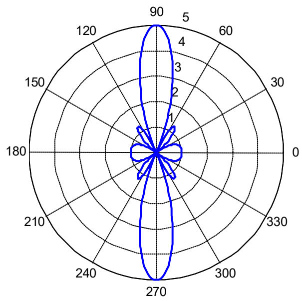
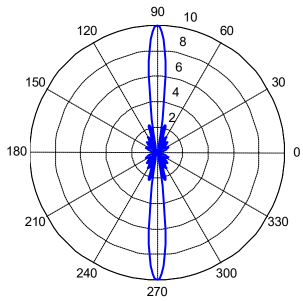
EPFL Examples d=0.5λ α=0







N=3 N=5 N=10

Antennas

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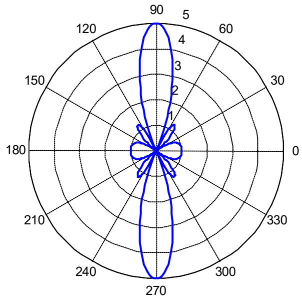
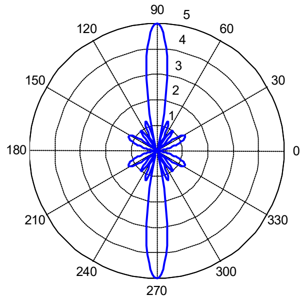
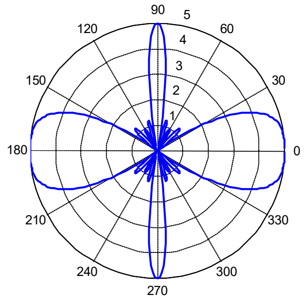
Examples : N=5, α=0



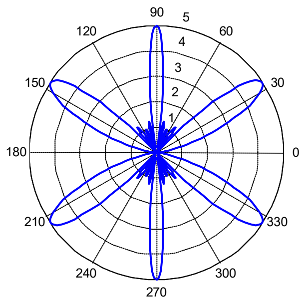


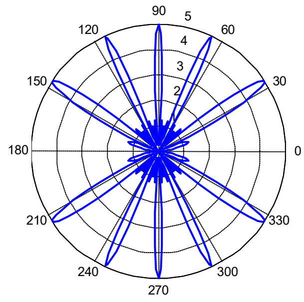


d=0.5λ d=0.8λ d=λ

Antennas

Examples : N=5, α=0





d=1.2λ d=2.4λ

Antennas

What happens?

The radiation of an equiamplitude array with linear phase shift is given by

$$NAF(\theta,\varphi) = \left| \frac{\sin\left(N\frac{\psi}{2}\right)}{N\sin\left(\frac{\psi}{2}\right)} \right|$$

Which is max for ψ= 0

$$\psi = kd\cos\theta_{\max} + \alpha = 0 \pm 2n\pi$$

Antennas

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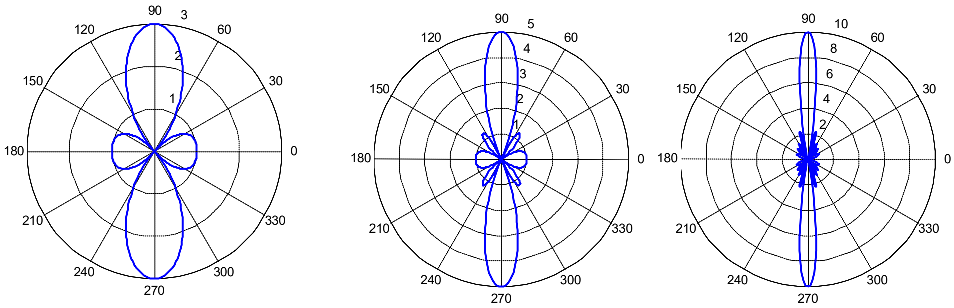
Examples d=0.5λ α=0



N=3 N=5 N=10

Antennas

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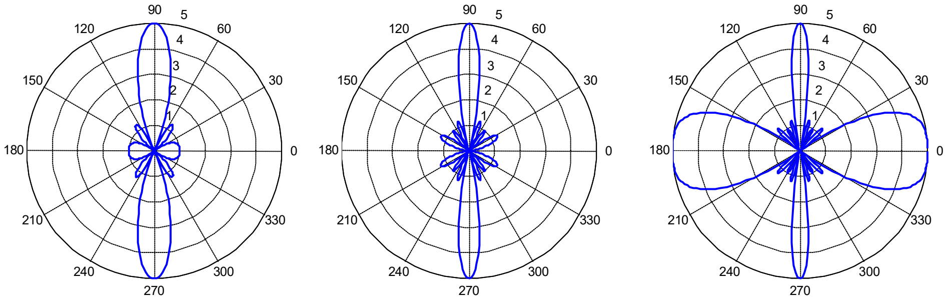
Examples : N=5, α=0



$d=0.5\lambda$ $\quad$ $d=0.8\lambda$ $\quad$ $d=\lambda$

Antennas

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Grating lobes

$$\psi = kd\cos\theta_{max} + \alpha = 0 \pm 2n\pi$$

In the case when α= 0 $\quad \psi = kd\cos\theta_{max} = 0 \pm 2n\pi$

If $kd<2\pi$, there is only one solution, $\theta_{max}=\pi/2$ ou $-\pi/2$

If $2\pi<kd<4\pi$, two solutions, $\theta_{max}=\pi/2$ ou $-\pi/2$ et, $\cos\theta_{max}=2\pi/kd$ ou $-2\pi/kd$

These are grating lobes

etc.

Antennas

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Array with steerable main beam

$$\psi = kd\cos\theta_{max} + \alpha = 0$$
$$\alpha = -kd\cos\theta_{max}$$

Broadside:

$\alpha=0 \ et \ \theta_{max} = \frac{\pi}{2}$

Endfire:

$\alpha = \pm\ kd \ et \ \theta_{max} = \pm\pi$

Antennas

Array with steerable main beam

$$\psi = kd\cos\theta_{max} + \alpha = 0 \pm 2n\pi$$
$$\alpha \in [-\pi;\pi]$$

1st case : main lobe, ψ=0

$$\alpha = -kd\cos\theta_{max}$$
$$\cos\theta_{max} = \frac{-\alpha}{kd} = \frac{-\alpha\lambda}{2\pi d}$$

Antennas

Array with steerable main beam

$$\cos\theta_{\max} = \frac{-\alpha}{kd} = \frac{-\alpha\lambda}{2\pi d} \quad ; \quad \alpha \in [-\pi;\pi]$$

$$if \quad d \geq \frac{\lambda}{2}$$

$$\left|\frac{\alpha\lambda}{2\pi d}\right| \leq 1 \quad and \quad \theta_{\max} \;\; exist\ whatever\ \alpha$$

Antennas

Array with steerable main beam

$$\cos\theta_{\max} = \frac{-\alpha}{kd} = \frac{-\alpha\lambda}{2\pi d} \quad ; \quad \alpha \in [-\pi;\pi]$$

$$if \quad d < \frac{\lambda}{2}$$

$$\left|\frac{\alpha\lambda}{2\pi d}\right| \leq 1 \quad for \;\; \alpha \in \left[-\frac{d}{2\pi\lambda};\frac{d}{2\pi\lambda}\right] and \;\; \theta_{\max} \quad exists$$

$$\left|\frac{\alpha\lambda}{2\pi d}\right| > 1 \quad for\ \alpha \in \left[-\pi;-\frac{d}{2\pi\lambda}\right[\cup \left]\frac{d}{2\pi\lambda};\pi\right]$$

$$and \;\; \theta_{\max} \quad does\ not\ exist$$

Antennas

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Grating lobes

$$\psi = kd\cos\theta_{\max} + \alpha = 0 \pm 2n\pi$$

$$\alpha \in [-\pi;\pi]$$

The firs grating lobe is the solution for $\psi=2\pi$

$$\psi = kd\cos\theta_{\max} + \alpha = 2n\pi$$

$$\cos\theta_{\max} = \frac{2\pi-\alpha}{kd}$$

$$\theta_{\max}\ exists\ if\ \left|\frac{2\pi-\alpha}{kd}\right| \le 1$$

$$thus\ if\ kd \ge |2\pi-\alpha|\ which\ is\ equivalent\ to\ \frac{d}{\lambda} \ge \left|1-\frac{\alpha}{2\pi}\right|$$

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Examples :

Broadside case: $\alpha = 0$

There will be grating lobes for $\frac{d}{\lambda} \ge \left|1-\frac{\alpha}{2\pi}\right| = 1$

Endfire case: $\alpha = \pi$ ou $-\pi$

There will be grating lobes for $\frac{d}{\lambda} \ge \left|1-\frac{\alpha}{2\pi}\right| = \frac{1}{2}$

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Beamwidth

The array factor is given by

$$NAF(\theta,\varphi)=\left|\frac{\sin\left(N\frac{\psi}{2}\right)}{N\sin\left(\frac{\psi}{2}\right)}\right|$$

It is largest for $\psi = kd\cos\theta_{max} + \alpha = 0 \pm 2n\pi$

It equals 0 for $\psi = kd\cos\theta_0 + \alpha = \pi \pm 2n\pi$

From this, the beamwidth is easily obtainable

Antennas

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Schelkunoff array synthesis

Array snthesis

- Analog to filter synthesis, with the angle replacing the frequency
- Depends in all generality on θ et φ
- For arrays with a symmetry of revolution, ot depends only on θ
- There are many synthesis methods among which Schelkunoff's method

Antennas

Schelkunoff synthesis for an equidistant linear array

The following variable is inroduced

$$w = e^{jkd\cos\theta}$$

The array factor becomes

$$AF(\theta) = \sum_{n=0}^{N-1} I_n w^n$$

This polynome has N-1 complex toots and can be written as

$$AF(\theta) = I_{N-1}(w - w_1)(w - w_2)(w - w_3)\ldots(w - w_{N-1})$$

Antennas

For each linear array with N euidistant elements we can set N-1 directions θ_n for whih we have a zero radiation (AF=0).

Schelkunoff synthesis for an equidistant linear array

We select the roots of the polynome: $w_n = e^{jkd\cos\theta_n}$

θ_n is a direction for which we want a zero in the radiation pattern.

The array factor is constructed as

$$AF(\theta) = I_{N-1}(w-w_1)(w-w_2)(w-w_3)\ldots(w-w_{N-1})$$

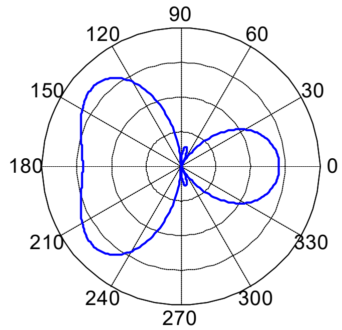
We come back to the polynomial form $AF(\theta) = \sum_{n=0}^{N-1} I_n w^n$

Which grieves the excitation factors I_n, and finally $AF(\theta) = \sum_{n=0}^{N-1} I_n e^{nkd\cos\theta}$

Antennas

Example: 3 element array seprated by half a wavelength

1: zero at θ=60°et θ=90°



a(1)=1 ; a(2)=-1-j ; a(3)=j

Antennas

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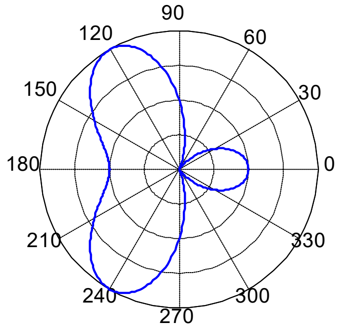
Example: 3 element array seprated by half a wavelength

1: 2 zeroes at θ=60°



a(1)=1 ; a(2)=-2*j ; a(3)=-1

Antennas

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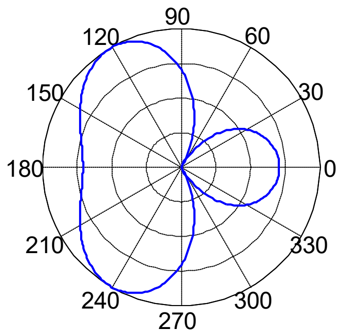
Example: 3 element array seprated by half a wavelength

1: zero at θ=60°and w=0

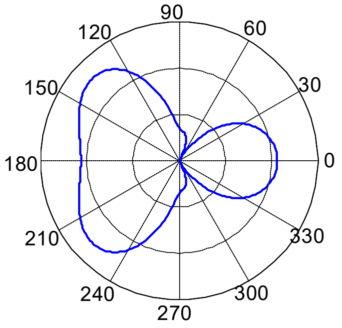


a(1)=1 ; a(2)=-j ; a(3)=0

Antennas

Example: 3 element array seprated by half a wavelength

1: zero at θ=60°and w=2



a(1)=1 , a(2)=-(2+j) ; a(3)=2*j

Antennas

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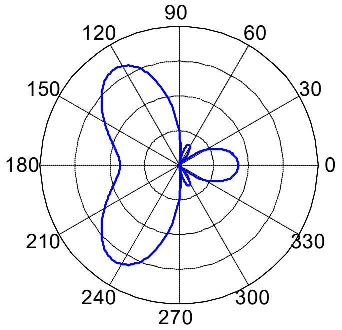
Example: 3 element array seprated by half a wavelength

1: zeroes at θ=45°and θ=80°



a(1)=1 ; a(2)= -0.2492 - 1.3146j ; a(3)= -0.9306 + 0.3659j

Antennas

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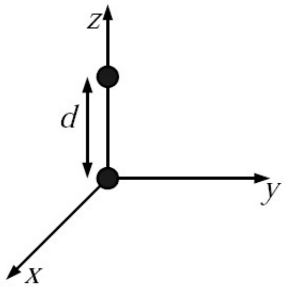
The binomial array

- In an equiamplitude array, the sidelobe level will never go below -13.3 dB
- We can show that for an equispaced array, the minimum sidelobe level is achived for the binomial

The binomial array

Trivial case: N=2, equiamplitude excitation



$$AF(\theta) = 1 + e^{j\psi} = 1 + e^{j(kd\cos\theta + \alpha)} = 1 + w$$

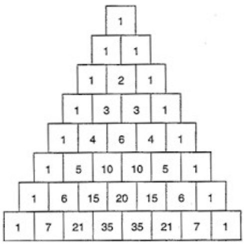
1+w is equal to zer0 only for w=-1,
Thus for kd cos θ + α = $\pm\pi$
If α=0, 1+w=0 has a solution only when kd>π
Now if an expression has no zero for a certain range of variables, then the powers of these expression also do not have a zero for this same range

The binomial array

The array factor is thus constructed in the following way

$$AF(\theta) = (1+w)^{N-1} = \sum_{n=0}^{N-1} \binom{N-1}{n} w^n$$

The array excitation coefficient are given by Newton's binomial expressions of by Pascal's triangle



example: 5 elements spaced by $\lambda/2$, α=0

Excitation coefficients for an equiamplitude excitation:
1 1 1 1 1
Excitation coefficient for a binomial array:
1 4 6 4 1

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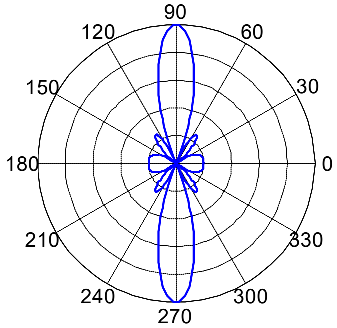
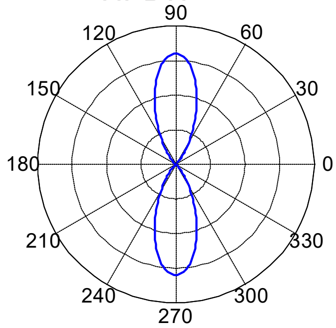
Dmax= 5
θ_{HPBW}= 20.8°.



equiamplitude

Dmax= 3.66
θ_{HPBW}= 30.3°.



Binomial

Antennas

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